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The energy density of "wound" fields in a toroidal universe

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Abstract: The observational limits on the present energy density of the Universe allow for a component that redshifts like $1/a^2$ and can contribute significantly to the total. We show that a possible origin for such a contribution is that the universe has a toroidal topology with "wound" scalar fields around its cycles.

Keywords: Cosmology of Theories beyond the SM, Beyond Standard Model.

Contents

1. Introduction

In this note we will describe a rather tight scenario which leads to a contribution to the present energy density of the universe, at the level of roughly one percent and redshifts like $1/a^2$, in addition to the cosmological constant and matter.

The script has several actors whose parts we describe next. The story starts by assuming a flat universe with the spatial topology of a torus, more specifically, this torus is a cube with opposite sides identified.

We assume that the universe undergoes a period of inflation which has the minimum amount of e-foldings necessary to solve the horizon and flatness problem. Any period of inflation beyond that number of e-foldings amount would inevitably dilute the $1/a^2$ contribution well below one percent of the present energy density. The physical size of the torus at the onset of inflation will be assumed to be such that the size of this torus today reaches the lower bound obtained by the search for "circles in the sky" [1]. There are other bounds on the topology of the universe from the CMB fluctuations [2] as well as from galactic methods [3, 4].

The matter content has, in addition to the inflaton and conventional low energy states, at least three massless angle-valued fields, θ_i, similar to axions.

We will show in what follows, that with these actors and some additional assumptions which will be introduced further in the text, we will be lead to the existence of a component in the energy density that scales like $1/a^2$. Such a contribution to the Friedmann equation mimicks a negative spatial curvature term but unlike the curvature term, it does not modify the relation between angular and the radial distance from what it is in flat space [5]. One crucial assumption will be the existence of a non-trivial topological configuration for the angles, θ_i.

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One might wonder why anybody would entertain such extreme assumptions. A partial answer is that the present data, as best we can tell, does not exclude a contribution of a percent or so to the present energy density that could arise from such an extravagant construction and therefore warrants some exploration.

We will begin by clearly stating the various assumptions and then describe how they can lead to a contribution of the order of a percent to the total energy density of the present universe.

2. The spatial torus and the topology of fields

We will present in this section the setup that leads to a one percent contribution to the present energy density of the universe which exhibits the $1/a^2$ behaviour. In what follows, the universe is assumed to be spatially flat with the topology of a torus.

For simplicity, we will take the torus to be a cube with opposite sides identified. The coordinate distance between opposite sides of the cube is L. We will denote the three cycles of the torus by $x_i = \frac{L}{2\pi}\chi_i$, where the χ_i are angles.

The next essential ingredient involves the existence of three minimally coupled scalar fields, φ_i where $i = 1, 2, 3$ which span in field space, a cube with opposite sides identified. This cube[1] in field space has linear size f, in our script f (like L) is a parameter available for tuning.

$$\varphi_i = \frac{f}{2\pi}\theta_i \tag{2.1}$$

where the θ_i are angle valued fields.

These scalar fields are assumed to couple to matter through their derivatives[2] and that no potential is generated for these fields, i.e. the lagrangian and the quantum dynamics that it generates, is invariant under the following transformations:

$$\theta_i \to \theta_i + c_i \tag{2.2}$$

where c_i are (space-time independent) constants. These scalars can be thought of as massless axions. In order to protect the isotropy of the microwave background as will become clear next, we impose an exact Z_3 symmetry, which acts on the three φ_i.

The equations of motion for the scalar fields in a Friedmann-Robertson-Walker (FRW) flat universe are:

$$\ddot{\theta}_i + 3\frac{\dot{a}}{a}\dot{\theta}_i - \frac{\nabla^2\theta_i}{a^2} = 0 \tag{2.3}$$

where the FRW metric we use is:

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j\,. \tag{2.4}$$

[1]If one allows for different values for the "decay constants", f_i, then as will be seen a little further, in order to respect the limits on the isotropy of the microwave background, the sizes, L_i, of the cycles of the spatial torus must satisfy $L_1/w_1 f_1 = L_2/w_2 f_2 = L_3/w_3 f_3$.

[2]The assumption that the φ_i are minimally coupled scalars ensures that the gravitational coupling of these scalars to matter is also through derivatives.

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Next, we choose as solution of equations (2.3), a topologically non-trivial configuration for the scalar fields:

$$\theta_i = w\chi_i \tag{2.5}$$

where w is the same winding number for each angular field around its respective cycle.

This very special choice of a background solution for the θ_i, will ensure that we comply with the isotropy of the microwave background within the stringent limit of one part in 10^5. Indeed, the energy-momentum tensor, $T_{\mu\nu}$, associated to this solution has the spatial components T_{mn} proportional to δ_{mn}.

The energy density, ρ_w, stored in this configuration of scalar fields is:

$$\rho_w = \frac{\nabla_i \varphi_j \nabla^i \varphi_j}{2a^2} = \frac{3w^2 f^2}{2L^2 a^2}\,. \tag{2.6}$$

This energy density redshifts like the spatial curvature contribution to the Friedmann equation, however it does not alter the relation between angular and radial distances from their flat space behaviour.

The same dependence of the energy density on the scale factor can be obtained for an ideal fluid with the following equation of state:

$$P = -\frac{1}{3}\rho\,. \tag{2.7}$$

It is amusing to note that this equation lies at the boundary between equations of state that lead to an accelerating universe and the ones that lead to the deccelerating case. However, in our script as will become clear, the energy stored in the "wound" scalar fields has never been the dominant component of the energy density of the universe. Therefore that component of the energy density does not affect the leading characteristics of the expansion of the universe.

Next, we will discuss what the requirements on the various parameters are, in order for this energy density to survive to the present day and contribute at best one percent of the total energy density.

Before that, we want to briefly address the reader that may wonder whether these massless scalars generate long range forces that would experimentally rule them out. A simple answer to such concern, is to require that these scalars not couple directly to the low energy degrees of freedom, except through gravity. In addition, we should also stress that even in the absence of such restriction, these scalars couple derivatively with a "decay constant" that is close to the Planck scale, as will be seen next, and are therefore not expected to affect static forces between bulk matter at macroscopic distances.

3. Constraints on parameter space

3.1 The requirements on the parameters related to the torus and scalars

There are various bounds on the spatial size of the universe when the universe is assumed to have the topology of a torus. The most stringent bound, to our knowledge, comes from the absence of “circles in the sky” [1] and requires the physical size of the universe to be no smaller than about six times the size of our present horizon.

$$La_0 \geq 24 \text{ Gpc} \sim 5 \times 10^{61} l_P \tag{3.1}$$

where $l_P = (\hbar G/c^3)^{1/2}$ is the Planck length.

We can now proceed to estimate the energy density, ρ_w stored today in the “wound” scalar fields as a function of the parameter f, “the decay constant” of the θ_i.

$$\rho_w = \frac{3w^2 f^2}{2L^2 a^2} \leq \frac{3w^2 f^2}{5 \times 10^{123} {l_P}^2} \tag{3.2}$$

which gives a contribution to the density of the universe today

$$\frac{\rho_w}{\rho_{cr}} = \left(\frac{f\, w}{2 \times 10^{19} \text{GeV}}\right)^2 . \tag{3.3}$$

A one percent contribution to the present energy density corresponds to $f \sim 2 \times 10^{18}$ GeV. Though, this decay constant is close to the Planck scale, it doesn't invalidate the use of classical gravity to describe the evolution of the universe. Indeed the contribution to the Ricci scalar from this form of energy is given by:

$$R\, l_P^2 \sim 24\pi \left(\frac{l_P}{La(t)}\right)^2 \left(\frac{w\, f}{1.2 \times 10^{19} \text{GeV}}\right)^2 . \tag{3.4}$$

Therefore as long as the physical size of the universe exceeds $10 l_P$, the scalar curvature is safely subplanckian.

It is worth reminding the reader that we imposed earlier a restriction on parameter space by choosing the same “decay constant”, f , for the θ_i to insure the isotropy of the microwave background.

There are also various other requirements on the “decay constant”, f, the size of the torus, L, as well as on the parameters related to inflation: the energy density during inflation Λ_I, the number of e-foldings, N_e, ... which will be discussed next.

3.2 The requirement on the parameters coming from inflation

In the presence of this additional form of energy, the onset of inflation depends on the energy density stored in the scalar fields to become smaller than the energy density, Λ_I, which sets the energy density scale of inflation.

$$\rho_w(t_{\text{beginning inflation}}) \ll \Lambda_I\, . \tag{3.5}$$

This constraint can be expressed as a function of the N_e and the reheating temperature, T_R,

$$\rho_w(t_0) \ll \Lambda_I e^{-2N_e} \left(\frac{T_0}{T_R}\right)^2 . \tag{3.6}$$

Another important constraint comes from requiring that the fluctuations generated during inflation, in the energy density of the scalar field (in the presence of their non-trivial topological background) not overwhelm the scale invariant fluctuations produced by

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the inflaton. The perturbations in the wound fields ($\delta\theta_i$) together with the perturbations on the inflaton field ($\delta\phi$) will generate a perturbation on the stress energy density. In turn, these perturbations will generate a perturbation in the metric (Ψ). To relate this latter variable to first two, we should construct the linear combination of these quantities that is conserved outside the horizon. It is given by [6, 7]

$$\zeta = \frac{-ik_i \delta T_i^0 H}{k^2(\rho + P)} - \Psi \,. \tag{3.7}$$

One can separately analyze the contributions to δT_i^0 from the two types of fields.

$$\delta T_w{}^0{}_i = -\frac{w f^2}{L}\frac{d\delta\theta_i}{dt} \tag{3.8}$$

$$\delta T_\phi{}^0{}_i = -ik_i \frac{d\phi}{dt}\delta\phi \,. \tag{3.9}$$

Since $|\frac{d\delta\theta_i}{dt}| = \frac{H}{\sqrt{2}}|\delta\theta_i|$ at horizon crossing, the fluctuations around the wound fields, if dominant, will generate a non scale invariant power spectrum. Both the inflaton field and the wound fields are scalars and will have the same quantum perturbations.

$$\langle \delta\phi(\mathbf{k}, t)\delta\phi(-\mathbf{k}, t)\rangle = f^2 \langle \delta\theta_i(\mathbf{k}, t)\delta\theta_i(-\mathbf{k}, t)\rangle \tag{3.10}$$

Hence the condition to avoid violations of scale invariance is that:

$$\rho_w \ll (P_\phi + \rho_\phi) = \left(\frac{d\phi}{dt}\right)^2 . \tag{3.11}$$

This condition, in turn, ensures that the contribution to Ψ from the wound fields is negligible at sub-horizon and crossing horizon scales.

4. Experimental information

In this section we explore the observational constraints on the various inflationary parameters discussed in the previous section. In addition, we also discuss the extent to which the $1/a^2$ contribution to the energy density could be present today. Recent WMAP data [8] yields values

$$r(k_0 = 0.002\,\mathrm{Mpc}^{-1}) \,<\, 0.90 \tag{4.1}$$

$$A(k_0 = 0.002\,\mathrm{Mpc}^{-1}) \,=\, 0.75^{+0.08}_{-0.09} \tag{4.2}$$

where r is the relative amplitude of the tensor to scalar modes

$$r \equiv \frac{\Delta_h^2(k_0)}{\Delta_R^2(k_0)} \tag{4.3}$$

and $A(k_0)$ and $\Delta_R^2(k_0)$ are related through

$$\Delta_R^2(k_0) \simeq 2.95 \times 10^{-9} A(k_0) \tag{4.4}$$

Standard slow roll analysis to the first order gives:

$$\Delta_R^2(k_0) = \frac{V/M_P^4}{24\pi^2\epsilon_V} \tag{4.5}$$

$$r = 16\epsilon_V \tag{4.6}$$

where the slow roll parameter ϵ_V is defined by:

$$\epsilon_V \equiv \frac{M_P^2}{2}\left(\frac{V'}{V}\right)^2 \tag{4.7}$$

$$M_P \equiv (8\pi G)^{-1/2} = 2.4 \times 10^{18}\,\mathrm{GeV}\,. \tag{4.8}$$

During inflation

$$H^2 \simeq \frac{V}{3M_P^2} \tag{4.9}$$

$$3H\dot{\phi} \simeq -V' \tag{4.10}$$

so the WMAP data corresponds to

$$\dot{\phi} \simeq 2 \times 10^{-34}\frac{V}{\mathrm{GeV}^2} \tag{4.11}$$

$$V < (3.1 \times 10^{16}\,\mathrm{GeV})^4\,. \tag{4.12}$$

In the previous section we have obtained the requirement (3.11):

$$\rho_w < \dot{\phi}^2 \tag{4.13}$$

Assuming that the modes of the size of present horizon left the horizon during inflation N_e e-foldings before the end of the inflation, which in our scenario is the beginning of inflation, and also assuming maximally efficient reheating, we obtain:

$$f < 1.2 \times 10^{-47} e^{-N_e} V^{3/4} La_0/\mathrm{GeV} \tag{4.14}$$

(To make this estimate, we use $\rho_w < \frac{1}{10}\dot{\phi}^2$.) In particular, taking $La_0 \sim 24\,\mathrm{Gpc}$, $V \sim (3 \times 10^{16}\,\mathrm{GeV})^4$ and $N_e = 60$ we obtain the limit

$$f < 10^{19}\,\mathrm{GeV}\,. \tag{4.15}$$

There are additional constraints from the observational data which can potentially limit the contribution of $1/a^2$ term to the present energy density. In particular, recent high-redshift supernova measurements [9] seem to allow flat Universe with e.g. $\Omega_{\mathrm{matter}} \simeq 0.27$, $\Omega_\Lambda \simeq 0.53$ and $\Omega_w \simeq 0.20$, within the 95% confidence level (Ω_w being the contribution of $1/a^2$ term to the present energy density). A considerable Ω_w would also have an effect on the age of the Universe. In particular, for a flat Universe with $\Omega_{\mathrm{matter}} = 0.27 \pm 0.04$ [8], $\Omega_w = 0.2$, $\Omega_{\mathrm{radiation}} = 10^{-4}$ and $\Omega_\Lambda = 1 - \Omega_{\mathrm{matter}} - \Omega_{\mathrm{radiation}} - \Omega_w$, using the HST Key Project [10] value of $H_0 = 72 \pm 3 \pm 7\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ one obtains the age of the Universe to be $13 \pm 2\,\mathrm{Gyr}$ which is in line with the present observational data [11, 12, 13, 14]. It is probable, that as suggested to us by E. Komatsu, the growth rate of density fluctuations and the mass function of collapsed objects will be useful to put more stringent constraints on Ω_w.

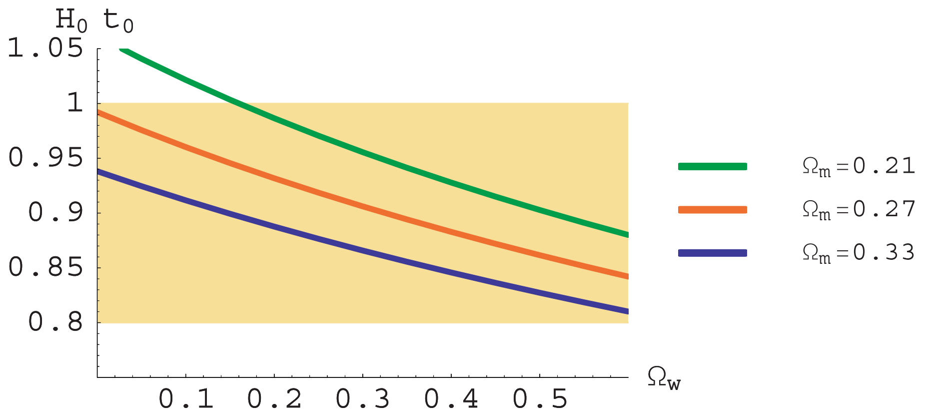

Figure 1: The effect of $\Omega_w = 1 - \Omega_m - \Omega_\Lambda$ on the age of the Universe. The light yellow region corresponds to the globular cluster age 12.7 ± 0.7 Gyr [12].

5. Conclusions

The experimental data seem to leave open the possibility of a contribution to the present energy that redshifts like $1/a^2$. We present in this paper an amusing and extremely constrained setup that cannot yet be excluded experimentally and does yield such a contribution to today's energy density.

There are, in addition, other known sources [15, 16, 17] for a component of the energy density that behaves like $1/a^2$. It is unclear, however, whether these sources would contribute in such manner today [18]. Our hope, is that the experiments will put firmer bounds on the existence of a $1/a^2$ contribution to the present energy density, irrespective of what sources such contribution.

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